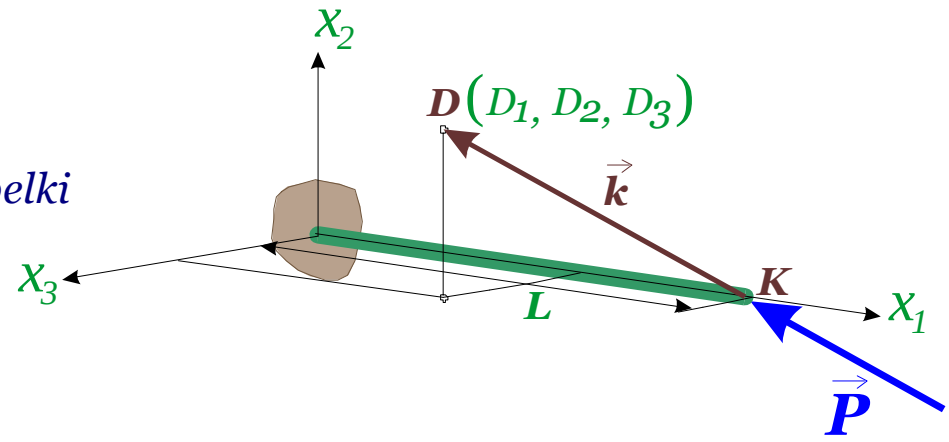
Grupa 1

Zad 1

$L := 7m \qquad P0 := 8kN \qquad b := 20cm \qquad h := 25cm \qquad g := 3cm$

$D := \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix} m$ - *współrzędne punktu przez który przechodzi kierunek siły*

$K := \begin{pmatrix} L \\ 0 \\ 0 \end{pmatrix}$ - *współrzędne punktu K, obciążonego końca belki*



$k := D - K$ - *wektor kierunkowy siły* $\qquad k = \begin{pmatrix} -4 \\ 5 \\ -2 \end{pmatrix} m$

$Lk := \sqrt{(k_1)^2 + (k_2)^2 + (k_3)^2} = 6.7082\,m$ - *moduł (długość) wektora kierunkowego*

$c := \frac{1}{Lk} \cdot k = \begin{pmatrix} -0.596285 \\ 0.745356 \\ -0.298142 \end{pmatrix}$ - *kosinusy kierunkowe wektora siły P*

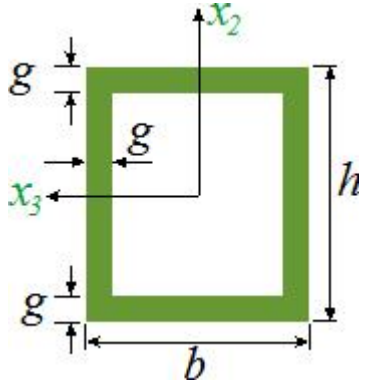
$P := P0 \cdot c$ *- składowe wektora siły* $P = \begin{pmatrix} -4.77 \\ 5.963 \\ -2.385 \end{pmatrix} \cdot kN$

$N := P_1 \qquad T2 := P_2 \qquad T3 := P_3$

$N = -4.77028 \cdot kN \qquad T2 = 5.96285 \cdot kN \qquad T3 = -2.38514 \cdot kN$

$M2 := -T3 \cdot L \qquad M3 := T2 \cdot L$

$M2 = 1.6696 \times 10^{1} \cdot kN \cdot m \qquad M3 = 4.17399 \times 10^{1} \cdot kN \cdot m$



$h1 := h - 2g \qquad b1 := b - 2g$

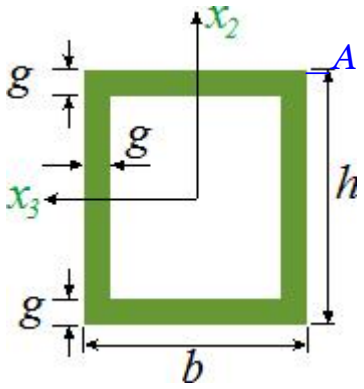
$A := h \cdot b - h1 \cdot b1 = 234 \cdot cm^2$

$J3 := \frac{b \cdot h^3}{12} - \frac{b1 \cdot h1^3}{12} = 1.80395 \times 10^{4} \cdot cm^4$

$J2 := \frac{h \cdot b^3}{12} - \frac{h1 \cdot b1^3}{12} = 1.2322 \times 10^{4} \cdot cm^4$

Naprężenia w punkcie A

$y := \frac{h}{2}$ $z := \frac{-b}{2}$ $a2 := h$ $a3 := b$



$S3 := 0$ $S2 := 0$

$$\sigma 11 := \frac{N}{A} - \frac{M3 \cdot y}{J3} + \frac{M2 \cdot z}{J2} = -42.676 \cdot MPa$$

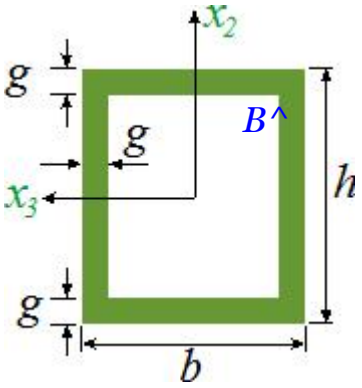
$$\tau 12 := \frac{T2 \cdot S3}{a3 \cdot J3} = 0.000 \times 10^{0} \cdot MPa$$

$$\tau 13 := \frac{T3 \cdot S2}{a2 \cdot J2} = 0.000 \times 10^{0} \cdot MPa$$

$$\sigma_{HMH} := \sqrt{\sigma 11^{2} + 3 \cdot \left(\tau 12^{2} + \tau 13^{2}\right)} = 42.676 \cdot MPa$$

Naprężenia w punkcie B

$y := \frac{h}{2} - g \quad z := \frac{-b}{2} + g \quad a2 := 2g \quad a3 := 2g$



$S3 := b \cdot g \cdot \frac{(h - g)}{2} \qquad S3 = 660 \cdot cm^3$

$S2 := g \cdot h \cdot \frac{(b - g)}{2} \qquad S2 = 637.5 \cdot cm^3$

$\sigma 11 := \frac{N}{A} - \frac{M3 \cdot y}{J3} + \frac{M2 \cdot z}{J2} = -31.67 \cdot MPa$

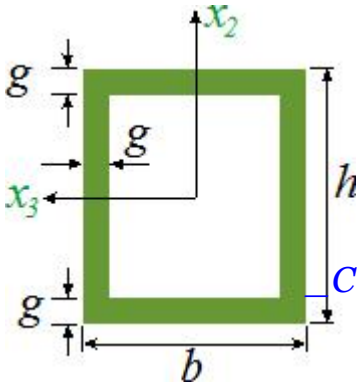
$\tau 12 := \frac{T2 \cdot S3}{a3 \cdot J3} = 3.636 \times 10^{-1} \cdot MPa$

$\tau 13 := \frac{T3 \cdot S2}{a2 \cdot J2} = -2.057 \times 10^{-1} \cdot MPa$

$\sigma_{HMH} := \sqrt{\sigma 11^2 + 3 \cdot \left(\tau 12^2 + \tau 13^2\right)} = 31.678 \cdot MPa$

Naprężenia w punkcie C

$y := \frac{-h1}{2}$ $z := \frac{-b}{2}$ $a2 := h$ $a3 := 2g$



$S3 := b \cdot g \cdot \frac{(h - g)}{2}$ $S3 = 660 \cdot cm^3$

$S2 := 0$ $S2 = 0 \cdot cm^3$

$\sigma 11 := \frac{N}{A} - \frac{M3 \cdot y}{J3} + \frac{M2 \cdot z}{J2} = 8.228 \cdot MPa$

$\tau 12 := \frac{T2 \cdot S3}{a3 \cdot J3} = 3.636 \times 10^{-1} \cdot MPa$

$\tau 13 := \frac{T3 \cdot S2}{a2 \cdot J2} = 0.000 \times 10^{0} \cdot MPa$

$\sigma_{HMH} := \sqrt{\sigma 11^2 + 3 \cdot (\tau 12^2 + \tau 13^2)} = 8.252 \cdot MPa$

Zad. 2

$E := 17GPa$ $L := 6m$ $b := 20cm$ $h := 25cm$ $g := 4cm$ $\mu 0 := 0.699156$

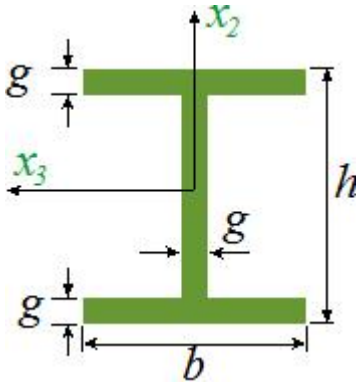
$\mu := 2.0$ $Lw := \mu \cdot L$ $b1 := b - g$ $h1 := h - 2 \cdot g$

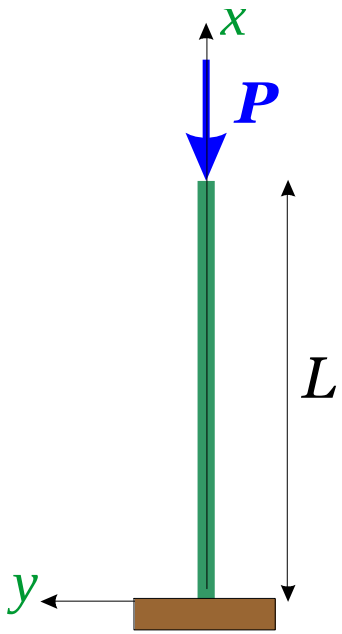
$$J3 := \frac{b \cdot h^3}{12} - \frac{b1 \cdot h1^3}{12} \qquad J3 = 1.949 \times 10^{-4}\, m^4$$

$$J2 := \frac{g \cdot b^3}{6} + \frac{h1 \cdot g^3}{12} \qquad J2 = 5.424 \times 10^{-5}\, m^4$$

$$J := \min(J2, J3) = 5.424 \times 10^{-5}\, m^4$$

$$Pkr := \frac{\pi^2 E \cdot J}{Lw^2} = 63.198 \cdot kN$$

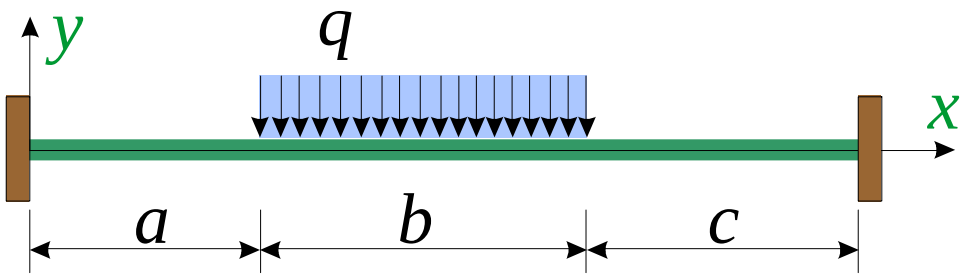


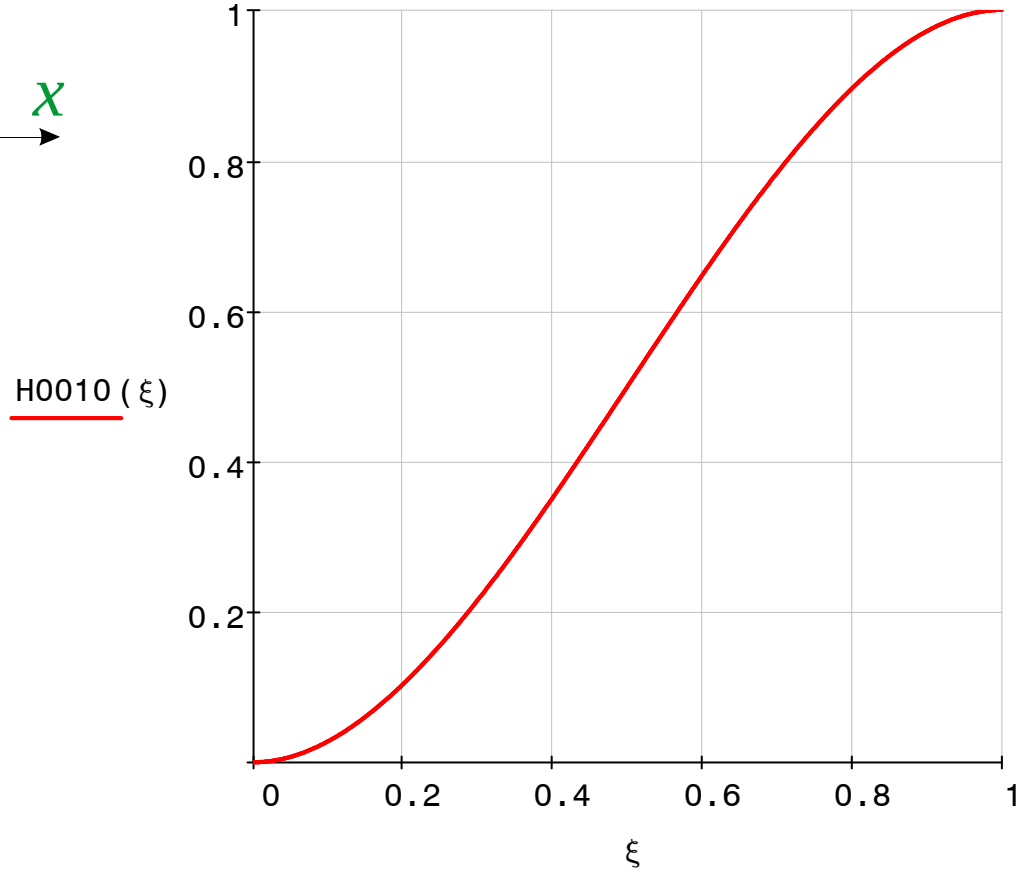


Zad. 3

$q := 5\,\frac{kN}{m}$ $a := 3m$ $b := 4m$ $c := 2m$ $L := a + b + c$

$\xi 1 := \frac{a}{L} = 0.333333$ $\xi 2 := \frac{a + b}{L} = 0.777778$ $H0010(\xi) := \xi^2 \cdot (3 - 2 \cdot \xi)$





Równanie pracy wirtualnej

$$R \cdot 1 - q \cdot L \cdot \int_{\xi 1}^{\xi 2} H0010(\xi)\, d\xi = 0$$

$$R := q \cdot L \cdot \int_{\xi 1}^{\xi 2} H0010(\xi)\, d\xi = 11.550069 \cdot kN$$

Definicja wielomianów Hermite'a dla belki obustronnie sztywno zamocowanej

$$H1000(\xi) := 1 - 3\cdot\xi^2 + 2\cdot\xi^3 \qquad H0010(\xi) := \xi^2\cdot(3 - 2\cdot\xi)$$

$$H0100(\xi) := \xi\cdot\left(1 - 2\cdot\xi + \xi^2\right) \qquad H0001(\xi) := -\xi^2\cdot(1 - \xi)$$

Definicja wielomianów Hermite'a dla belki zamocowanej przegubowo:

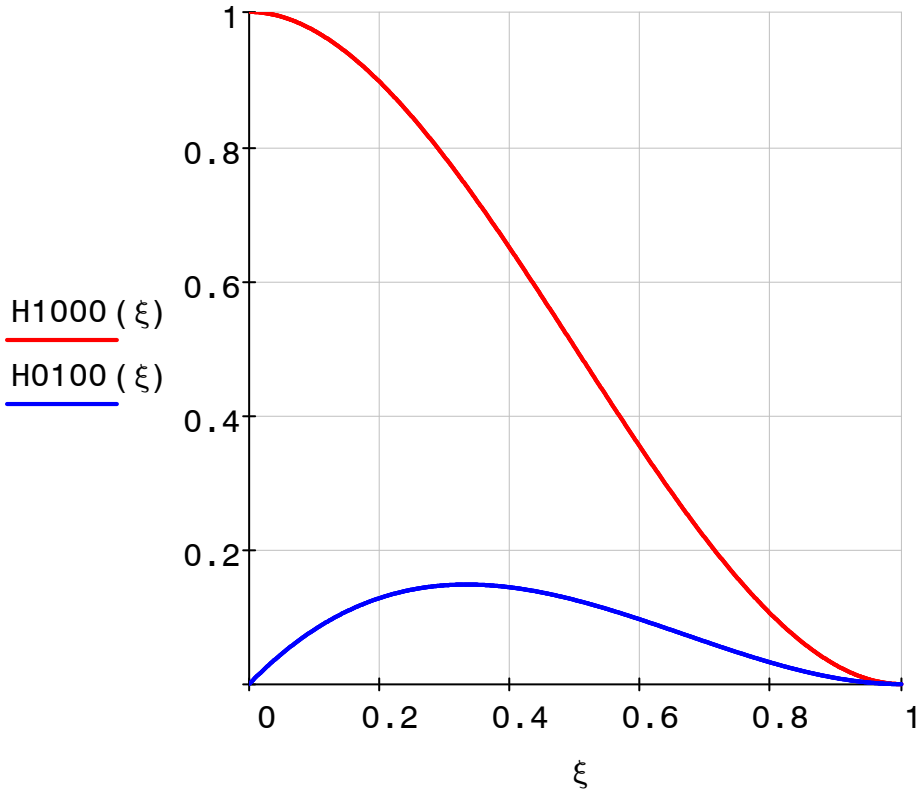
na prawej podporze - G(ξ) *na lewej podporze - K(ξ)*

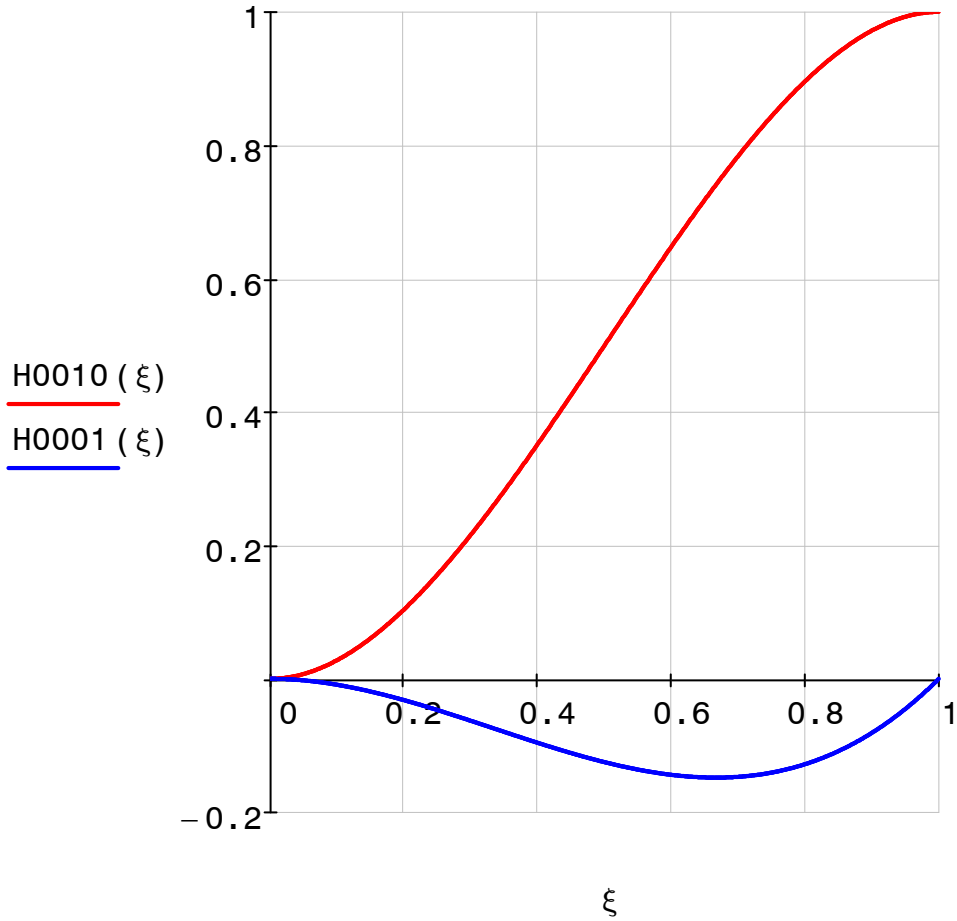
$$G1000(\xi) := 1 - \frac{3}{2}\cdot\xi^2 + \frac{1}{2}\cdot\xi^3 \qquad K1000(\xi) := \frac{1}{2}\xi^3 - \frac{3}{2}\xi + 1$$

$$G0100(\xi) := \frac{\xi}{2}\cdot\left(2 - 3\cdot\xi + \xi^2\right) \qquad K0010(\xi) := \frac{\xi}{2}\cdot\left(3 - \xi^2\right)$$

$$G0010(\xi) := \frac{\xi^2}{2}\cdot(3 - \xi) \qquad K0001(\xi) := \frac{\xi}{2}\cdot\left(\xi^2 - 1\right)$$

Wykresy wielomianów Hermite'a dla belki obustronnie sztywno zamocowanej





Wykresy wielomianów Hermite'a dla belki zamocowanej przegubowo:
na prawej podporze - G(ξ) *lewej podporze - K(ξ)*